E Trigonometry

E3

Understand and use the sine, cosine and tangent functions; their graphs, symmetries and periodicity.

Know and use exact values of sin and cos for $0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}, \pi$ and multiples thereof, and exact values of tan for $0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \pi$ and multiples thereof.

Teaching guidance

Students should be able to:

- understand and use vertical asymptotes of a tangent graph
- carry out simple transformations (as given in section B9) of the graphs of the sine, cosine and tangent functions.

At A-level combinations of transformations may be used.

Note: radians will not be required at AS.

E Trigonometry

E5

Understand and use $\tan\theta \equiv \dfrac{\sin\theta}{\cos\theta}$.

Understand and use $\sin^2\theta + \cos^2\theta \equiv 1$; $\sec^2\theta \equiv 1 + \tan^2\theta$ and $\operatorname{cosec}^2\theta \equiv 1 + \cot^2\theta$

Assessed at AS and A-level

Teaching guidance

Students should be able to:

- use $\tan\theta \equiv \dfrac{\sin\theta}{\cos\theta}$ to solve equations or find exact values
- use $\sin^2\theta + \cos^2\theta \equiv 1$ to solve equations or find exact values.

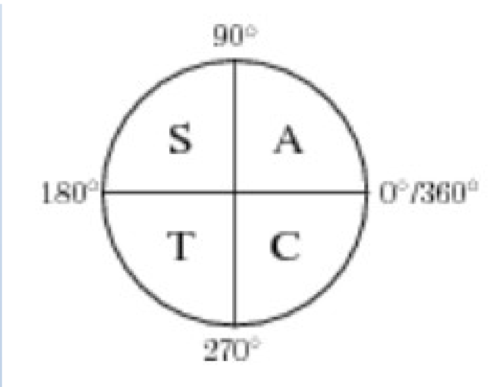
3.1 Sine, cosine and tangent

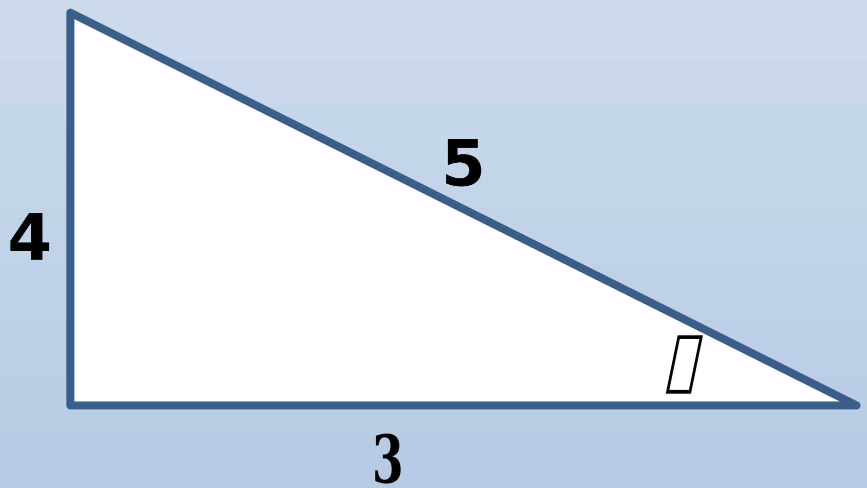
Example 4a

a Given that $\cos\theta = -\frac{3}{5}$ and that θ is reflex, find the value of $\sin\theta$.

b Given that $\sin\alpha = \frac{2}{5}$ and that α is obtuse, find the exact value of $\cos\alpha$.

eflex so sin will be negative.





$$\therefore \sin\theta = -\frac{4}{5}$$

Pythagoras, the opposite side is 4.

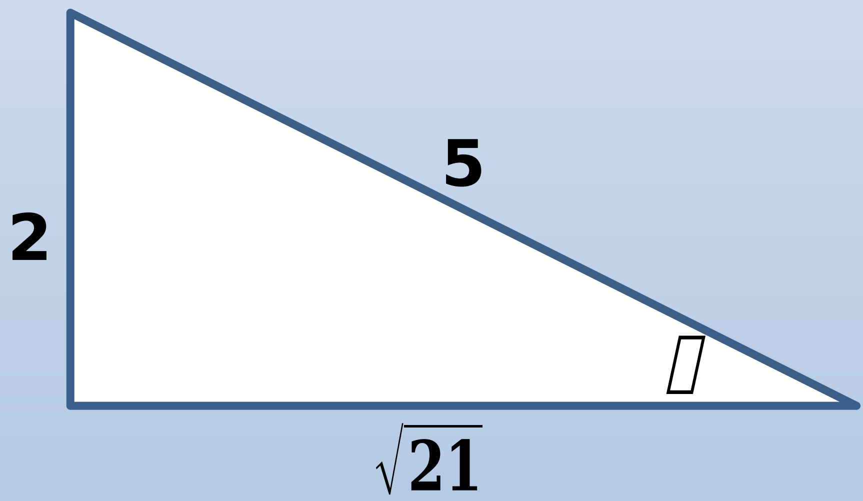
3.1 Sine, cosine and tangent

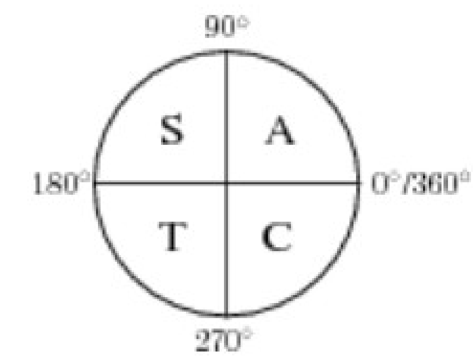
Example 4b

a Given that $\cos\theta = -\frac{3}{5}$ and that θ is reflex, find the value of $\sin\theta$.

b Given that $\sin\alpha = \frac{2}{5}$ and that α is obtuse, find the exact value of $\cos\alpha$.

s obtuse so cos will be negative.





$$\therefore \cos\alpha = -\frac{\sqrt{21}}{5}$$

3.1 Sine, cosine and tangent

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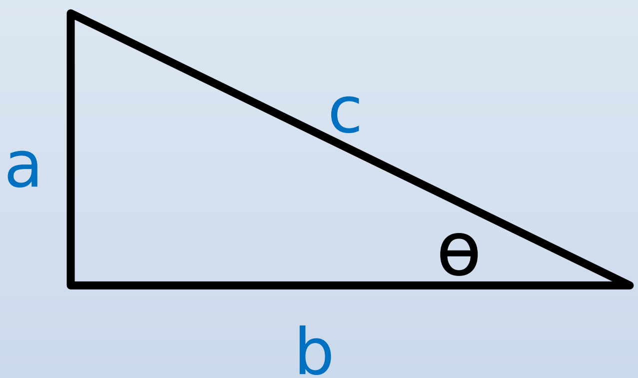

$$\sin \theta = \frac{a}{c}$$

$$\cos \theta = \frac{b}{c}$$

$$\tan \theta = \frac{a}{b}$$

Pythagoras' theorem for right-angled triangles is:

$$a^2 + b^2 = c^2$$

Dividing by c^2 gives:

+ =

Or:

+ = 1

$$\boxed{\sin^2\theta + \cos^2\theta \equiv 1}$$

3.1 Sine, cosine and tangent

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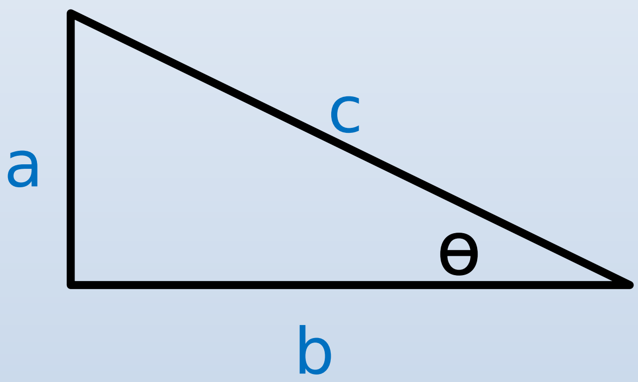


$$\sin \theta = \frac{a}{c}$$

$$\cos \theta = \frac{b}{c}$$

$$\tan \theta = \frac{a}{b}$$

$$\tan \theta = \frac{a}{b}$$

$$\tan \theta = \frac{\frac{a}{c}}{\frac{b}{c}}$$

$$\boxed{\tan \theta \equiv \frac{\sin \theta}{\cos \theta}}$$

3.1 Sine, cosine and tangent

Example 5

Use a trig identity to find sin θ and tan θ as surds, given that θ is acute and cos θ =

is acute so all sin , cos and tan are positive...

3.1 Sine, cosine and tangent

Example 5

Use a trig identity to find sin θ and tan θ as surds, given that θ is acute and cos θ =

$$\sin \theta = \sqrt{\frac{2}{3}}$$

3.1 Sine, cosine and tangent

Example 6

Prove that

LHS:

= RHS as required

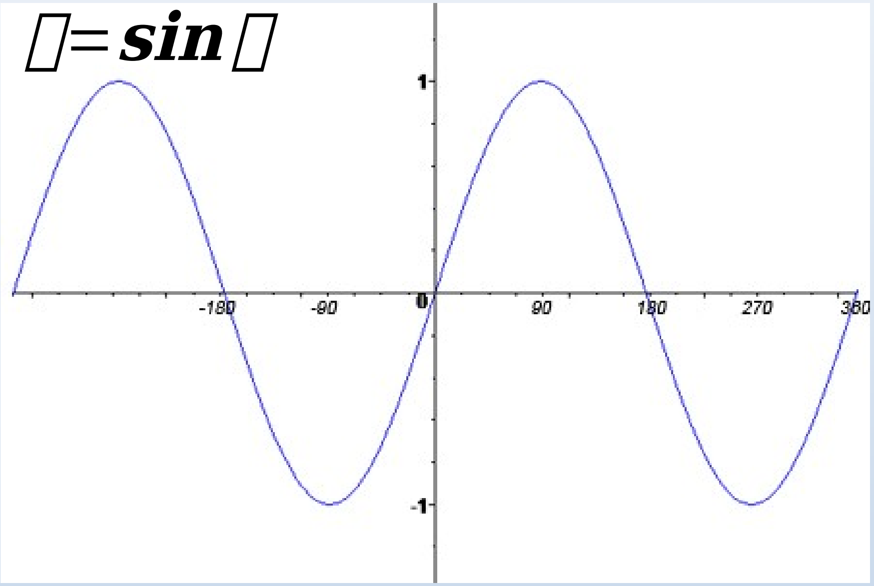
3.1 Sine, cosine and tangent

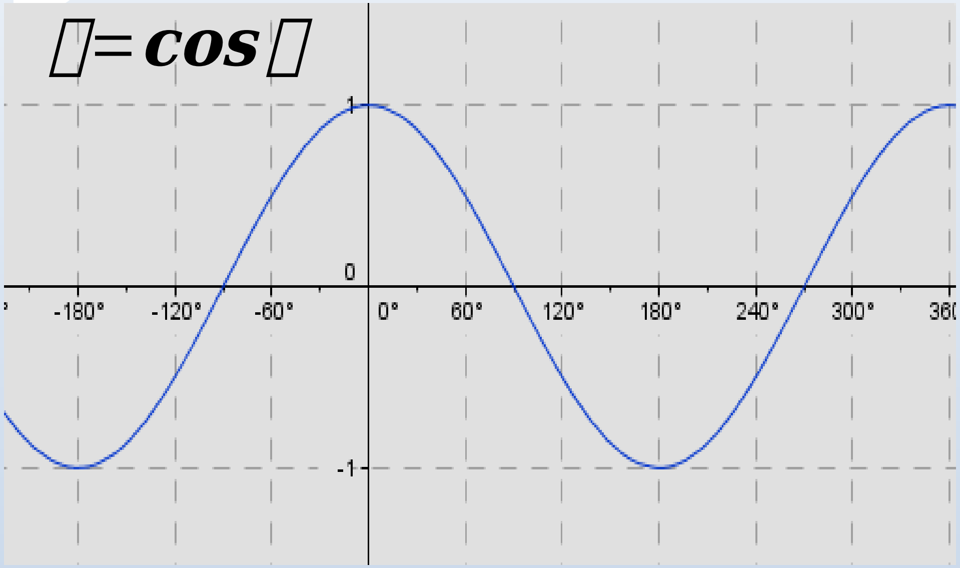
Example 7

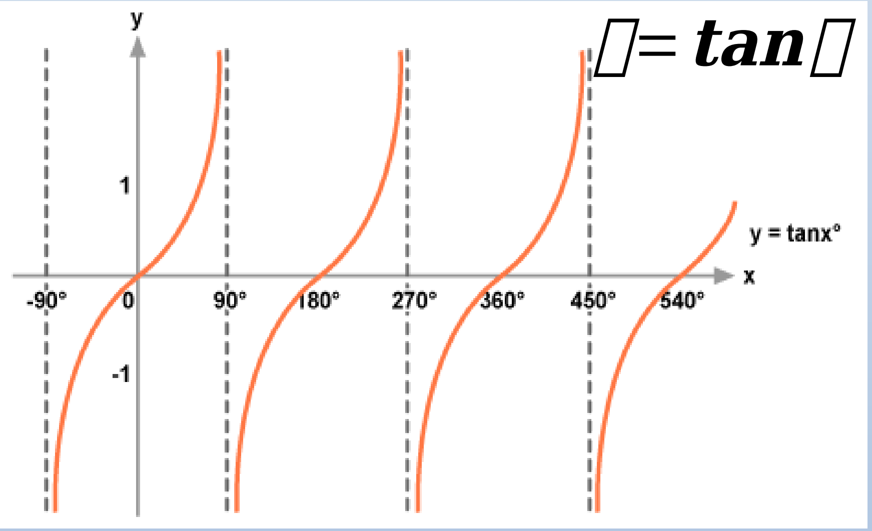
Prove that $\dfrac{\cos^4\theta - \sin^4\theta}{\cos^2\theta} \equiv 1 - \tan^2\theta$

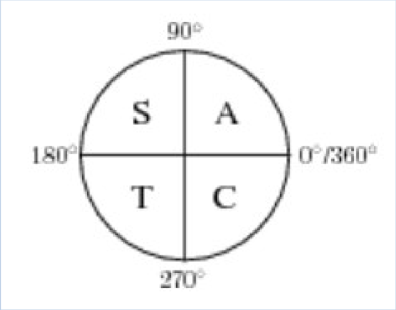
¿ $1 - \tan^2$ □

3.1 Sine, cosine and tangent









$$\boxed{\tan\theta \equiv \frac{\sin\theta}{\cos\theta}}$$

$$\boxed{sin^2\theta + cos^2\theta \equiv 1}$$